The linguistic roots of multiplication

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Abstract:
It is a well-established fact, confirmed by various experiments, that preschoolers, human infants, and even non-human primates can perform intuitive addition and subtraction. Much less evidence has been put forth testifying that children are capable of multiplicative operations on sets before receiving formal training. What makes evidence of intuitive multiplication hard to obtain is that in the visual and auditive domains multiplication is often indistinguishable from repeated addition. This paper claims that multiplication operations are routinely performed by children prior to schooling; they are encoded by syntactic means in such doubly quantified sentences as the Hungarian Három maci is két autóval játszik 'Three teddy bears (each) are playing with two cars', denoting a situation with six cars. The paper reports on three experiments testing Hungarian preschoolers' strategies of interpreting such sentence. The experiments show that (i) Hungarian preschoolers have access to the multiplicative reading of doubly quantified sentences; they not only recognize but can also actively compute the product of multiplication. (ii) At the same time, children's strategies of interpreting the scope order of doubly quantified sentences are less constrained than those of adults; their selection of the multiplier and the multiplicand may depend on the linear order of the quantified expressions, their thematic prominence relation, and/or the visual representation/grouping of the sets denoted by them. (iii) Of the multiplicative and collective readings of a doubly quantified sentence, children choose the multiplicative reading when pragmatics makes the collective reading implausible, i.e., they treat the collective interpretation as default – presumably because its computation imposes a lesser load on their cognitive system.

Keywords: multiplication, intuitive arithmetic, syntax, quantification, scope interpretation

1. Introduction
It is a well-established fact, confirmed by various experiments, that preschoolers, human infants, and even non-human primates can perform intuitive addition and subtraction. Much less evidence has been put forth testifying that children are capable of multiplicative operations on sets before receiving formal training. McCrink & Spelke (2010) have demonstrated that preschoolers can perform doubling, quadrupling and increasing by 2.5 of large approximate numerosities, and a line of research has shown up multiplication in animals’, infants’, and children’s recognition of proportional relations (Gallistel, 1990; McCrink & Wynn, 2007; Schlottmann & Tring, 2005). What makes evidence of intuitive multiplication hard to obtain is that in the visual and auditive domains multiplication is often indistinguishable from repeated addition.

This paper will argue that multiplication operations are routinely performed by children prior to schooling; they are encoded by syntactic means in such doubly quantified sentences as the Hungarian Három maci is két autóval játszik 'Three teddy bears (each) are playing with two cars'. We will report on three experiments testifying that Hungarian preschoolers accept
such a sentence as a true statement about a situation involving three teddy bears and six cars. What is more, they can also actively set up situations representing the meanings of such sentences, computing the product of multiplication themselves. The three experiments reveal what syntactic and pragmatic clues induce children to assign multiplicative readings to doubly quantified sentences, and how they decide which quantifier is the multiplier and which one is the multiplicand (in linguistic terms: what clues elicit the distributive interpretation of doubly quantified sentences, and how the scope order of the quantifiers is determined).

2. Psychological background

A large number of experiments have demonstrated beyond doubt that addition and subtraction form part of our toolkit of intuitive arithmetic; not only preschoolers but also infants and even non-human primates can compute the outcomes of additive and subtractive operations over visually presented sets of elements – cf. Wynn (1992), Dehaene (1997 ch. 2), McCrink & Wynn (2004), Barth, La Mont, Lipton, & Spelke (2005), Flombaum, Junge, & Hauser (2005), Cantlon and Brannon (2007), etc. Preschoolers have been shown to be capable of addition across modalities, adding up the numerosities of arrays of visual and auditory items (Barth, La Mont, Lipton & Spelke, 2005; Barth, La Mont, Lipton, Dehaene, Kanwisher, & Spelke, 2006; Barth, Beckman & Spelke, 2008). Animals’, infants’, and children’s ability to add and subtract is based on their ability to represent approximate numerical magnitudes in an analog fashion (cf., e.g., Xu & Spelke, 2000; Slaughter, Kamppi & Paynter, 2006; Barth, Beckmann & Spelke, 2008; Cordes and Brannon, 2009).

It is less obvious whether children – let alone infants or non-human primates – relying on analog magnitude representations of approximate numerosities are capable of multiplicative operations on sets. What makes the testing of multiplication difficult is that in the visual domain the product of multiplication can in most cases be derived by repeated addition, as well. Studies have sought to circumvent this problem by testing the ability of animals, children, and mathematically untrained adults to detect ratios, i.e., specific proportional relations, between quantities and numerosities. Thus it has been pointed out that foraging animals are sensitive to differences in reward rates, and quickly adjust to rate changes to maximize their reward (cf. Gallistel, 1990; Gallistel, Gelman, & Cordes, 2005; Cordes et al., 2007), which suggests that they perform computation multiplying the average amount of food observed or obtained per food encounter with the number of food encounters per unit time (Gallistel, 1990, p. 382). The same has been demonstrated for adults with no formal schooling, e.g., Brazilian fishermen (Nunes et al., 1993).

Infants have also been shown to be sensitive to ratios. McCrink and Wynn (2007) found that six-month old infants habituated to a series of slides displaying large, changing numbers of objects of two types in a constant ratio noticed when their ratio changed.

A number of studies have pointed out preschoolers’ ability to detect proportional relationships. Schlottmann & Tring (2005) argued that 6-year-old children choose between sure gain and gamble by calculating the ratio of risk and the amount at risk. Boyer et al. (2008) found that although children have difficulties solving proportional reasoning problems involving discrete units until 10 to 12 years of age, they can solve parallel problems involving continuous quantities by 6 years of age. Barth, Baron, Spelke and Carey (2009) investigated whether kindergarteners can identify halving and doubling over numerical and continuous values. They found that the children were capable of halving, but the results were inconclusive as regards doubling. In McCrink & Spelke’s (2010) experiment, 5-7-year-old children were given a task requiring a scalar transformation (doubling, quadrupling, or increasing by 2.5) of large approximate numerosities, presented as arrays of objects. In all conditions, children were able to represent the outcome of the transformation at above-chance levels, even on the earliest training trials. The authors claim that „the success of children on
these experiments cannot be explained by a process of repeated addition. First, the children were able to successfully multiply by a factor of 2.5 and a factor of 4.0. In order to use repeated addition, children would need to mentally represent 8 arrays (for Times 2.5) and 5 arrays (for Times 4); both of these amounts exceed the number of arrays even adults can hold in working memory (Halberda et al., 2006). Second, even if they were somehow able to use repeated addition with this many arrays, this account predicts that performance would be lowest on the Times 2.5 condition, which is not the case.” Their performance in discriminating the outcome of multiplication from a comparison array was sensitive not to the absolute difference of the two amounts, but to their ratio, which indicates that they were relying on the approximate number system.

In comparison to the large amount of robust evidence testifying that addition and subtraction form part of the biologically determined toolkit of humans (and even of higher animals), the evidence for the availability of multiplication for animals, infants, and kindergarteners appears to be scant. However, this may be due to the fact that most experiments have been designed to test multiplication in the visual domain, where it is hard to distinguish from repeated addition. We propose a different testing ground: natural language sentences containing two quantifiers entering into scope interaction. Such sentences, e.g., *Both kids would like three cookies*, provide a multiplier (in this case, 2) and a multiplicand (in this case, 3), and their interpretation includes the calculation of the product of multiplication by the listener.

Our experiments have aimed at finding out whether preschoolers, who have not received any arithmetic training, can understand such sentences, i.e., whether they can perform the syntactically encoded multiplicative operations to be computed in the course of semantic interpretation.

The processing of this sentence type has already been tested among English-speaking preschoolers by Musolino (2009). (We were not aware of this study when we designed and started our experiments in 2010.) The phenomenon we intend to show up and account for is very similar to what Musolino has found, namely: “children readily accept sentences like *Three boys are holding two balloons*, even if the total number of balloons [shown in the picture that is being talked about], six, is different from the one explicitly mentioned in the sentence, two” (Musolino 2009, p. 36). Hence our hypothesis that the interpretation of doubly quantified sentences involves intuitive multiplication carries over to English, and presumably to other languages, as well. At the same time, the syntactic encoding of this type of multiplicative operations also has elements that are specific to the grammars of individual languages, which necessitates language-specific investigations. We have chosen Hungarian for our experiments because Hungarian is known to be a language where the syntactic encoding of logical operations has become grammaticalized (i.e., standardized) to a larger extent than in other languages with a thoroughly described grammar of quantification.

3. Linguistic background

A sentence with two quantifiers such as (1) can have at least three different meanings, those paraphrased in (1a), (1b), and (1c).

(i) Three teddy bears are playing with two cars.
   a. 'There are three teddy bears, each of which is playing with two (possibly different) cars.'
   b. 'There are two cars, each of which three (possibly different) teddy bears are playing with.'
   c. 'There are three teddy bears and two cars, and the former are playing with the latter.'

Readings (i)a and (i)b are so-called distributive readings. Under reading (i)a, where *three* has wide scope and *two* has narrow scope, in other words, *three* has scope over *two* (i.e., $3 > 2$),
the situation involves three teddy bears, and two cars distributed to each of them, i.e., altogether six cars. (More precisely, it involves three teddy bears and up to six cars, given that the two cars assigned to each of the three teddy bears may partially or fully coincide.) Under reading (i)b, where two has wide scope (i.e., $3 < 2$), the situation involves two cars and (up to) six teddy bears. Under reading (i)c, where both quantifiers have independent scopes, the situation involves three teddy bears and two cars altogether. The latter meaning is called collective or cumulative depending on whether the group of teddy bears is playing with the group of cars, or different members of the set of teddy bears are playing with different members of the set of cars. Since this distinction is not relevant from our present perspective, it will be ignored, and reading (i)c will simply be referred to below as ‘collective’.

In English, quantifier scope is not systematically encoded in overt syntax. Sentences with two or more quantifiers are usually ambiguous; the selection of the preferred interpretation can be affected by various syntactic factors such as the linear order, the grammatical function (e.g., subject versus object status), and the thematic (e.g., agent or patient) role of the quantified constituents. In the case of (i), the most unmarked reading is presumably that in (i)c. The other two readings can be elicited by special, marked means, e.g, by the insertion of each:

(ii) Three teddy bears are each playing with two cars.

Pragmatic considerations can also facilitate one reading or the other. Thus, in the case of (iii), the distributive/multiplicative reading, whereas in the case of (iv), the collective reading is more likely.

(iii) Every boy was eating an apple.
(iv) Every boy was watching a football game.

In Hungarian, quantifier scope marking has mostly been grammaticalized, i.e., surface syntax disambiguates scope (Hunyadi, 1986; É. Kiss, 1987; 1991; 2002; 2011; Szabolcsi, 1994; 1997; Szabolcsi & Brody, 2003; Surányi, 2002; 2006; etc.). The three interpretations of (i) are expressed by three different sentences in Hungarian:

(v) Három maci is két autóval játszik.
three teddy bear two car-with plays
'There are three teddy bears, each of which is playing with two (possibly different) cars.'

(vi) Két autóval is három maci játszik.
two car-with three teddy bear plays
'There are two cars, each of which three (possibly different) teddy bears are playing with.'

(vii) Három maci játszik két autóval.
three teddy bear plays two car-with
'There are three teddy bears and two cars, and the former are are playing with the latter.'
In Hungarian, the primary means of encoding quantifier scope is word order - at least in the preverbal section of the Hungarian sentence, where most quantifiers are to be found. If a quantifier Q1 precedes a quantifier Q2 preverbally, Q1 has scope over Q2.\(^1\)

Syntactic theory predicts that a quantifier has scope over its sister constituent and her descendants in the tree diagram representing syntactic structure. Whereas in English, the structural representation encoding quantifier scope is a virtual structure, in the Hungarian sentence, quantifiers are overtly raised into structural positions corresponding to their scope. Compare the positions of a non-quantified subject and and a non-quantified object in Hungarian sentence structure (viii) with the positions of their quantified equivalents (ix). (The trace/original position of a moved constituent is marked by a symbol \(t\), which is coindexed with the moved element.)

(viii)  
\[
\begin{array}{c}
\text{TenseP} \\
\text{Tense} \\
\text{Játszanak}_i \\
\text{NP} \\
a \text{macik} \\
\text{V'} \\
\text{V} \\
t_i \\
\text{az autókkal} \\
\text{play} \\
\text{the teddybears} \\
\text{the cars-with} \\
\end{array}
\]

'The teddybears are playing with the cars.'

(ix)  
\[
\begin{array}{c}
\text{DistributiveP} \\
\text{minden macij} \\
\text{FocusP} \\
\text{két autóval}_k \\
\text{TenseP} \\
\text{Tense} \\
\text{játszik}_i \\
\text{NP} \\
t_j \\
\text{V'} \\
\text{V} \\
t_i \\
t_k \\
\end{array}
\]

every bear two car-with plays
'Every bear is playing with two cars.'

Focus Phrase and Distributive Phrase provide landing sites for different types of quantified expressions to be raised into scope positions - cf. Szabolcsi (1997). The landing site immediately dominated by DistributiveP is open to quantifiers that always induce scope dependency, i.e., which are inherently distributive, among them universal quantifiers such as

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\(^1\) This rule is exceptionless in the preverbal domain, and it also holds for most types of Q1 Verb Q2 orders. Ambiguity may arise in the case of two postverbal quantifiers, which is a rare sentence pattern, occurring, e.g., as a result of verb movement elicited in questions and focus constructions:

(i) Miért játszik két macik három autóval?
'why plays two teddy bear three car-with
'Why are two teddy bears playing with three cars?'

For most speakers, such sentences, too, are disambiguated by stress; the wide-scope quantifier is stressed, and the narrow-scope quantifier is destressed.
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minden 'every’. Hungarian also has an unmarked, regular means of turning numerically modified expressions into distributive quantifiers inducing scope-dependency: the particle is. Numerically modified expressions not supplied with is can assume scope by moving into the focus position immediately dominated by FocusP.

Is is a very common, multi-purpose particle; it has at least three different functions: it can act as an additive (x)a, an affirmative (x)b, or a distributive particle (x)c:

(x)  a. János is fel-emelte a zongorát.
  John up-lifted the piano
  'John, too, lifted the piano.'

b. János akarta fel-emelni a zongorát, és ő is emelte fel.
  John wanted up-lift-INFINITIVE the piano and he lifted up
  'It was John who wanted to lift the piano, and it was him, indeed, who lifted it.'

c. Két fiú is fel-emelte a zongorát.
  two boy up-lifted the piano
  'Two boys each lifted the piano.'

The additive and the affirmative uses of is require appropriate preceding contexts, hence when is occurs with a numerically modified expression in an out-of-the-blue sentence, adult speakers interpret it as a distributive particle. As such it causes the predicate phrase following the quantified expression to be distributed over the individuals in the set denoted by the quantified expression. Thus (x)c means that each of the boys lifted the piano separately. When the predicate phrase to be distributed contains another (narrower scope) quantified expression, the set denoted by the latter is also distributed, i.e., it is multiplied by the number of the individuals in the set denoted by the wider scope quantifier. Compare the interpretations of the following sentences:

(xi) a. Az előadáson hat sorban is 14 hallgató ült.
  the talk-at six row-in 14 listener sat
  'At the talk, there were 14 listeners sitting in each of six rows.'

b. Az előadáson 14 hallgató ült hat sorban.
  the talk-at 14 listener sat six row-in
  'At the talk, there were 14 listeners sitting in six rows.'

(xi)a clearly elicits multiplication; it means that the number of listeners was 6 times 14. (xi)b, on the other hand, involves no multiplication; it means that the number of listeners was 14. In a test involving 44 students of Budapest University of Technology and Economics, participants listened to these two sentences, and were asked after each one to answer the question „How many listeners were there?” 40 students (90%) answered „84” after the first sentence, and „14’ after the second sentence. (In fact, one of them answered „84 or more” –

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2 This sentence does not exclude the possibility of there being further rows with more or less than 14 listeners. This possibility could be excluded by using a universal determiner and a definite article:

(i) Az előadáson mind a hat sorban 14 néző ült.
  the talk-at all the six row-in 14 listener sat
  'At the talk, there were 14 listeners in each of the six rows.'
correctly, as discussed in footnote 2.) Of the remaining four students, three gave the same answer in both cases, one answered „84” and two answered „14”.

When interpreting doubly quantified sentences with low numbers, adults obviously rely on the multiplication table they memorized at school. But when the numbers are high, they compute the product approximately, presumably relying on the approximate number system.

Our experiments to be reported below demonstrate that Hungarian preschoolers, too, can access the distributive meaning of doubly quantified sentences. Rather than determining the scope order of the two quantifiers randomly, they have strategies for selecting the direction of scope dependency, even if these strategies are more flexible than those of Hungarian adults. The operation that they perform in the processing of such sentences is non-distinct from intuitive multiplication, and the assignment of wide scope and narrow scope in a doubly quantified sentence is non-distinct from selecting the multiplier and the multiplicand.

4. Research questions
We carried out three experiments, addressing the following questions:

Do preschoolers with no arithmetic training understand the distributive/multiplicative meaning of doubly quantified sentences? Do they judge sentences containing two quantifiers functioning as a multiplier and a multiplicand to be true as statements about visual representations of the product of multiplication?

We have also been interested in how children decide which quantifier of a doubly quantified sentence should have wide scope, and which one should have narrow scope, or, from a different perspective, how they select the multiplier and the multiplicand. We have wondered whether they determine scope order on the basis of the linear order of quantifiers, as Hungarian adults do, or they also use other strategies, based on different clues.

We found in pilot studies that children can interpret doubly quantified sentences involving is non-distributively, i.e., non-multiplicatively, as well. (The same was found about English doubly quantified sentences involving each by Musolino (2009).) In the third experiment, we wanted to test whether the multiplicative or the non-multiplicative reading of doubly quantified sentences is primary for preschoolers, and whether their choice of primary interpretation is affected by pragmatic conditions.

5. Experiment 1: truth value judgment
Experiment 1 aimed to find out whether Hungarian preschoolers understood the distributive reading of doubly quantified sentences; more precisely, whether they could compute the multiplication encoded in sentences containing a numerically quantified phrase modified by is taking scope over another numerically quantified phrase. The answer was provided by whether or not they could identify the product of multiplication represented visually, i.e., whether or not they judged a sentence such as Három maci is két autóval játszik ‘Three teddy bears are playing with two cars’ to be the true description of a picture showing three teddy bears and six cars, and/or the true description of a picture showing six teddy bears and two cars.

5.1. Method
The experiment involved the members of the „big kids’ group” in three kindergartens located in districts XI, XII, and XXII of Budapest. All three experiments were performed in all three kindergartens, on three separate occasions. The order of the three experiments was different at every location. The educational program of Hungarian kindergartens involves no arithmetic, i.e., the children tested had no formal training in arithmetic operations. We carried out no

3 We owe thanks to Érdi street Kindergarten, Táltos Kindergarten, and Halacska Protestant Kindergarten for helping our research in several ways.
earlier pilot studies in these kindergartens, and we had no test trials at the beginning of the experiments.

**Subjects:**
In the first experiment (Exp 1), 46 subjects, 27 (59%) males and 19 (41%) females participated. The mean age of the subjects was 6;5 years, SD=4 months.

The mean age for males was 6;7 years, SD=5 months, for females, 6;4 years, SD=3 months. There was no difference between the ages of males and females ($F$(1/44)=3.71, $p=0.06$). The mean ages of the children by kindergarten were as follows: Érdi street Kindergarten (EK): $M=6;6$, SD=2 months, Halacska Kindergarten (HK): $M=6;4$, SD=3 months, and Tältos Kindergarten (TK): $M=6;7$, SD=5 months ($F$(2/43)=2.51, $p=0.22$).

**Procedure:**
The child, the experimenter, and a helper were seated at a table in front of a laptop in a quiet room of the kindergarten. The helper had a hedgehog puppet on her hand. The experimenter explained that the child and the hedgehog would look at pictures on the computer screen together, and the hedgehog was going to tell the child what she saw in the picture. The hedgehog was old, and had weak eyes, hence she could not always see the picture properly. Each time when the hedgehog said something about a picture, the experimenter would ask the child whether the hedgehog was right or wrong. (In truth value judgment tasks, the sentences to be judged by children are uttered by a puppet instead of the experimenter because children are more willing to assume about a puppet than about an adult that she can be wrong.)

The child was presented 15 sentence–picture pairs (10 fillers and 5 test pairs) involving two quantifiers (listed in the Appendix). The pictures were photos of toys. Each test pair was preceded by two fillers. When presenting a picture, the experimenter said: „Let's listen to what the hedgehog sees in the picture”, and then asked the child if the hedgehog was right or wrong. The child received positive feedback from the experimenter after each answer („Well done”, etc.). The experimenter recorded each answer on a sheet. All the three experiments were videotaped.

The test started with two fillers, a picture showing a teddy bear and a red car, paired with a sentence that was obviously true (*A maci egy piros autóval játszik* 'The teddy bear is playing with a red car'), and a picture showing a bunny and three carrots, paired with a sentence that was obviously false (*A nyuszi két répát talált* 'The bunny found two carrots') – in order to make the child realize that the hedgehog is sometimes right and sometimes wrong. We did not have to exclude subjects for not being able, or willing, to perform the task.

The five test cases of picture–sentence combinations involved the following doubly quantified sentences:

(xii) Három maci is két autóval játszik.  
three teddy bear two car-with plays  
'Three teddy bears (each) are playing with two cars.'

(xiii) Két tornyot is három fiú épít.  
two tower-ACCUSATIVE three boy-NOMINATIVE builds  
'Two towers (each) are being built by three boys.' (Literally: 'Two towers (each), three boys are building.')</xcode>

(xiv) Két macinak is három autója van.  

4 The English translations of the Hungarian sentences contain a bracketed *each*; its bracketing is intended to express that it is more explicitly distributive than the multi-functional Hungarian *is* particle.
two teddy bear-DATIVE three car-POSSESSIVE is 'Two teddy bears (each) have three cars.'

These sentences describe situations which can, in principle, be interpreted either distributively or collectively. The verbs in (xii) and (xiii), meaning play and build, both denote activities that can be performed either together or individually, and the relation of possession expressed in (xiv) can also be either shared, i.e., collective, or individual, i.e., distributive. Sentences (xii) and (xiii) differ in the order of the S(ubject) and the O(bject). In the SOV (xii) the linear order of the quantified expressions corresponds to their functional ranking (subjects being more prominent than (prepositional) objects), and to their thematic ranking (agents being more prominent than patients). In (xiii), on the other hand, the linear order of the two quantifiers is the opposite of their order in the hierarchy of grammatical functions and thematic roles.

Both (xii) and (xiii) were paired with two pictures, one representing their multiplicative reading with direct scope (where the initial quantifier is the multiplier and the second quantifier is the multiplicand), and the other representing their multiplicative reading with inverse scope (where the initial quantifier is the multiplicand and the second quantifier is the multiplier). These sentence–picture combinations were intended to test whether preschoolers can associate doubly quantified sentences with multiplicative readings at all, and whether they only accept a multiplicative reading with direct scope, as Hungarian adults do. Sentence (xiv) was coupled with a picture showing its collective reading involving no scope dependency/no multiplication – in order to check whether the presence of the particle is blocks the collective reading of doubly quantified sentences for preschoolers (as it does for Hungarian adults). The sentence–picture combinations we tested are listed below. They are referred to by numbers marking their order of presentation in Experiment 1 (cf. the Appendix).
E1/6. Három maci is két autóval játszik.
'Three teddy bears (each) are playing with two cars.'
Picture showing inverse scope: 2 cars and 6 bears

E1/3. Két tornyot is három fiú épít.
'Two towers (each), three boys are building.'
Picture showing direct scope: 2 towers and 6 boys

E1/9. Két tornyot is három fiú épít.
'Two towers (each), three boys are building.'
Picture showing inverse scope: 3 boys and 6 towers
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E1/15. Két macinak is három autója van.
'Two teddy bears (each) have three cars.'
Picture showing independent scopes: 2 bears and 3 cars

These sentence–picture pairs represent the following 5 conditions (Sq = 'quantified subject'; Oq = 'quantified object'; Q1 > Q2 = 'Q1 has scope over Q2' (direct scope); Q1 < Q2 = 'Q1 is in the scope of Q2' (inverse scope)):

1. Sq > Oq Condition (E1/12): Subject first, Subject wide scope
2. Sq < Oq Condition (E1/6): Subject first, Object wide scope
3. Oq > Sq Condition (E1/3): Object first, Object wide scope
4. Oq < Sq Condition (E1/9): Object first, Subject wide scope
5. Q1, Q2 Condition (E/15): both quantifiers have independent scopes

The test sentence–picture pairs were spersed among the filler cases in an arbitrarily determined order (Conditions 3, 2, 4, 1, 5). The question in each case was whether the sentence was true about the picture presented.

5.2. Results:
Children judged the sentences in the five conditions as follows:

1. Sq > Oq Condition (E1/12): True=91%, False=9%
2. Sq < Oq Condition (E1/6): True=63%, False=37%
3. Oq > Sq Condition (E1/3): True=67%, False=33%
4. Oq < Sq Condition (E1/9): True=41%, False=59%
5. Q1, Q2 Condition (E/15): True=93%, False=7%

5 Here 'object' means prepositional object.
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![Figure 1](image)

Figure 1  The ratio between the True and False responses for the test sentences in Exp 1.

The difference between the mean numbers of True and False responses to the five picture-sentence test pairs by the subjects (M<sub>true</sub>=3.52 (SD=1.31), M<sub>false</sub>=1.48 (SD=0.90)) is significant (F(1/90)=55.78, p<0.001). The difference between the mean numbers of the responses to the five picture-sentence test pairs within males and within females (F<sub>male</sub>(1/52)=19.58, p<0.001; F<sub>female</sub>(1/36)=48.25, p<0.001) is also significant. However, there was no difference between the sexes in the number of their True responses and in the number of their False responses (F<sub>true</sub>(1/36)=0.86, p=0.36; F<sub>false</sub>(1/36)=0.87, p=0.35). The mean numbers of True responses given by the children of the three kindergartens did not differ, either (F(2/43)=0.63, p=0.54).

5.3. Discussion

The results indicate that preschoolers could assign distributive/multiplicative readings to doubly quantified sentences. In the majority of cases, they accepted a sentence containing the quantifiers 2 and 3 to be a true statement about a situation showing the product of multiplying 2 by 3 (or 3 by 2).

To be able to better assess the data obtained in the different conditions, we tested the sentence-picture pairs of experiment 1 with an adult control group, as well. Recall that adult Hungarian grammar has been claimed not to allow the inverse scope readings represented in Conditions 2 and 4 (cf. É. Kiss, 1991; 2002; Szabolcsi, 1994; 1997; Szabolcsi & Brody, 2003; Surányi, 2002; 2006; etc.). Furthermore, Hungarian adults have been claimed to interpret the particle <i>is</i> associated with a numerically modified constituent in an out-of-the-blue sentence as a distributive particle, i.e., to rule out the collective reading represented in Condition 5. In order to test these claims, we administered the sentence-picture pairs of Experiment 1 to 44 students of Budapest University of Technology and Economics. They were asked to grade on a scale from 1 to 5 how natural the sentence they heard sounded as a statement about the situation shown in the picture presented. The mean and median values we received basically confirm the predictions of Hungarian grammars:

**Adult control results:**
1. S<sub>q</sub> > O<sub>q</sub> Condition (E1/12):  Mean = 4.07 (SD: 1.21) median: 4.5
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2. \( S_q < O_q \) Condition (E1/6):  \( \text{Mean} = 1,55 \ (SD: \ 0,94) \) median: 1

3. \( O_q > S_q \) Condition (E1/3):  \( \text{Mean} = 3,11 \ (SD: \ 1,49) \) median: 3,5

4. \( O_q < S_q \) Condition (E1/9):  \( \text{Mean} = 1,75 \ (SD: \ 1,14) \) median: 1

5. Q1, Q2  Condition (E/15):  \( \text{Mean} = 2,22 \ (SD: \ 1,31) \) median: 2

That is, adults only accepted the direct scope readings (Conditions 1 and 3), and they preferred the variant where the initial, wide scope quantifier was the subject (Condition 1). They rejected the inverse scope interpretations (Conditions 2 and 4), and did not accept the collective reading of a doubly quantified sentence involving *is*, either (Condition 5).

The sentence–picture pair representing Condition 1 (\( S_q > O_q \)), involving a ‘subject first’ sentence associated with a picture showing its direct scope reading, was not only evaluated as most natural by adults but was accepted by 91% of the children participating in Experiment 1, as well. That is, nearly all Hungarian preschoolers understand the multiplication encoded in this sentence type, and can map it on a picture representing the product of multiplication. (Musolino (2009), who tested the direct scope interpretation of the sentence *Three boys are holding two balloons*, found a 78,1% acceptance rate among slightly younger English preschoolers (mean age 5;0).) Of the four Hungarian preschoolers who rejected this sentence–picture pair, two explained that the hedgehog was wrong because the picture showed 6 cars instead of 2. These children apparently could not compute the multiplication encoded by syntactic means. However, one of them could read the title on the experimenter’s folder, i.e., he seemed to receive some tutoring at home. His (and the other child’s) comments suggested to us that – noticing the test situation – perhaps they wanted to show off their counting skills, suppressing their intuitive interpretation of the sentence.

Whereas adult Hungarian speakers rejected the inverse scope reading of the subject-initial sentence in Condition 2, 63% of the children accepted it. This is an unexpected result, as the quantifier to which they assigned wide scope is less prominent in every respect (as regards its linear order, its grammatical function, and its thematic role) than the quantifier to which they assigned narrow scope. (In Musolino’s (2009) experiment only 28,1% of English preschoolers accepted the sentence *Three boys are holding two balloons* with an inverse scope interpretation.) We understood only after Experiment 2 what induced children to accept this sentence under its inverse scope reading. We return to this question in the discussion of Experiment 2.

The sentence–picture pair representing Condition 3 (\( O_q > S_q \)), involving an ‘object-first’ sentence paired with a picture showing its direct scope reading, was, again, acceptable to adults, but it proved to be less ideal than the ‘subject-first’ sentence with direct scope in Condition 1. Whereas the median rating of the latter was 4,5, the median rating of the former was only 3,5. Preschoolers, too, found Condition 3 difficult. Whereas in the 1st, \( S_q > O_q \) Condition they answered immediately, in the 3rd, \( O_q > S_q \) Condition many of them were thinking long before answering. As opposed to the 91% of subjects accepting the test sentence as true in the \( S_q > O_q \) Condition, only 67% of them accepted it in the \( O_q > S_q \) Condition. What makes sentence processing in the \( O_q > S_q \) Condition more difficult is the fact that here the linear order and scope order of the two quantified expressions is the opposite of the functional and thematic precedence relation between them, i.e., the functionally and semantically less prominent quantified expression stands first, and has wide scope. Three of the 15 subjects who rejected this sentence added that the hedgehog was wrong because the picture showed 6 boys, not 3.

The 4th, \( O_q < S_q \) Condition, assigning inverse scope to an object-initial sentence, was rejected by adults. Children also found it difficult to process; like in the \( O_q > S_q \) Condition,
they were thinking long before answering. In this case, the thematic and functional prominence relation of the two quantifiers is the opposite of their linear order, and the scope order matches their thematic/functional ranking (i.e., the non-initial, but thematically more prominent subject has wide scope). 41% of the children accepted this sentence–picture pair as true, i.e., 41% of them could interpret the sentence multiplicatively. The 27 children (59%) rejecting it include the 4 children (9%) who did not accept the multiplicative reading in the easiest S<sub>q</sub> > O<sub>q</sub> Condition, as well as 5 children (11%) who rejected both inverse scope conditions, and accepted both direct scope conditions, i.e., who seem to have already acquired adult-like competence.

The sentence in Condition 5 elicits a distributive reading in adult grammar (recall that the adults we tested rated it as 2 under a collective interpretation). Interestingly, 93% of the children accepted this sentence paired with a picture representing its non-distributive, collective reading. Two children indicated that *is* would require distribution/multiplication in this case, as well; a child rejecting the sentence, and another child accepting it mentioned that each of the teddy bears should have cars.

In sum, Experiment 1 demonstrated that the great majority of Hungarian preschoolers (91%) are capable of decoding and computing the multiplication operation encoded in doubly quantified sentences involving the particle *is*. At the same time, they also accept the non-distributive/non-multiplicative reading of such sentences. This ambiguity may be due to the lexical ambiguity of the particle *is*. Whereas in adult grammar the expected, primary function of a particle *is* modifying a numerically quantified expression is the marking of distributivity, for children, its additive particle function (corresponding to ’also’) may be more salient.

The proportion of the subjects who could map a doubly quantified sentence upon a picture showing the product of multiplication was 91% only when the picture represented direct scope, with the subject functioning as the clause-initial wide scope quantifier/multiplier. In the other three conditions, the acceptance of the distributive/multiplicative reading varied between 67% and 41%. We assume that the children who proved to be capable of computing multiplication in the S<sub>q</sub> > O<sub>q</sub> Condition, but refused the multiplicative reading in some other condition did so because they could not associate with the given sentence the scope order represented in the picture. The linguistic and psycholinguistic literature on quantifier interpretation has identified linear precedence, functional prominence, and thematic prominence as the major factors affecting scope interpretation. Actually, these factors cannot fully explain the distribution of acceptance rates in Conditions 1-4 (they cannot explain the fact that the S<sub>q</sub> < O<sub>q</sub> Condition was accepted by more subjects than the O<sub>q</sub> < S<sub>q</sub> Condition). In order to clarify the conditions determining scope interpretation, we carried out a further experiment.

6. Experiment 2: forced choice between pictures representing different scope orders

Experiment 1 has shown that nearly all Hungarian preschoolers are capable of deriving the multiplicative readings of sentences with two quantifiers. At the same time, it has also become clear that they have more than one strategy at their disposal to select the multiplier and the multiplicand, i.e., to determine which quantifier is to be assigned wide scope, and which one is to be assigned narrow scope. Whereas in adult grammar, Hungarian sentences containing two preverbal quantifiers are not ambiguous, their scope order being determined by their linear order, many subjects in Experiment 1 associated doubly quantified sentences with both scope orders. At the same time, the inverse scope reading associated with the doubly quantified sentence in Condition 4 was rejected by the majority, i.e., scope assignment cannot have been unconstrained.

Earlier studies on adults’ and children’s strategies of scope interpretation (among them Ioup, 1975; 1982; Micham et al., 1980; Fodor, 1982; Kurtzman & McDonald, 1983; Johnson-
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Laird, Byrne & Tabossi, 1989; Gillen, 1991; Tunstall, 1998; Lidz & Musolino, 2002; Lee, 2003; Musolino & Lidz, 2003; 2006) teased apart three crucial factors determining the scope order of two quantifiers: their precedence relation, their functional prominence relation (i.e., subject vs. object role), and their thematic prominence relation (i.e., their relative prominence in the hierarchy ‘agent > location > patient’). It was the role of these factors in the selection of children’s preferred scope interpretation that Experiment 2 was intended to test. Since in Hungarian there is no passive voice reversing the functional ranking of the agent and the patient, the distinction of the thematic and functional hierarchies did not seem to be crucial; hence we only tested the role of linear order and thematic ranking in children’s scope interpretation.

As this time we were not interested in the presence versus absence of certain readings but wanted to get to know children’s preferred choice of the two distributive interpretations accessible to the majority of them, we opted for a forced choice task instead of truth value judgements.

6.1. Method
Subjects:
41 subjects participated in Experiment 2, among them 22 males (54%) and 19 females (46%). They were recruited from the same three kindergartens as in Experiment 1. The mean age of the subjects was $M=6;6$, $SD=4$ months. The mean age for males was $M=6;7$, $SD=5$ months, for females was $M=6;4$, $SD=3$ months ($F(1/39)=4.46$, $p<0.05$). The ages of the children by kindergarten: EK$_{(n=14)}$: $M=6;6$, $SD=4$ months, HK$_{(n=16)}$: $M=6;4$, $SD=4$ months, TK$_{(n=11)}$: $M=6;8$, $SD=5$ months ($F(2/38)=1.17$, $p=0.32$).

Procedure:
The child, the experimenter, and a helper were seated at a table in a quiet room of the kindergarten. The helper had a hedgehog puppet on her hand. The experimenter told the child that they were going to play a game. The child and the hedgehog would be presented pairs of pictures. The hedgehog would say what she saw in one of the two pictures, and the child had to find out which of the two pictures the hedgehog was talking about.

The child was presented 20 pairs of pictures (drawings) – see Appendix 2. Each pair showed the direct and the inverse scope readings of a doubly quantified sentence. The two A5-size (148 mm x 210 mm) pictures were placed on the table side by side, in a previously fixed, randomly determined order. The experimenter told the child to look at both pictures carefully. After 4-5 seconds, the hedgehog uttered a sentence (with the helper avoiding looking at either of the pictures), and the experimenter asked the child which of the two pictures (s)he thinks the hedgehog spoke about. The child pointed at the one of the two pictures, and the experimenter recorded his/her choice on a sheet. After giving the child some positive feedback, the experimenter removed the pictures, and put the next pair on the table.

The 20 pairs included 8 test cases and 12 fillers. In the 8 test cases, 4 pairs of pictures were used. Each pair of pictures was shown twice, coupled with sentences which differed in the relative order of the two preverbal quantifiers. The 8 test sentences and the descriptions of the pairs of pictures coupled with them are listed below. The numbers correspond to the order of presentation in the experiment. (The left vs. right-hand side position of the drawings in each pair was determined arbitrarily in advance.)
E2/18. Két fiú is három tornyot épít.
   two boy three tower-ACCUSATIVE builds
   'Two boys (each) are building three towers.'
Left picture: 3 towers, 6 boys;    Right picture: 2 boys, 6 towers

E2/3. Három tornyot is két fiú épít.
   three tower-ACCUSATIVE two boy-NOMINATIVE builds
   'Three towers (each) are being built by two boys.' (Literally: 'Three towers each, two boys are building.')
Left picture: 3 towers, 6 boys;    Right picture: 2 boys, 6 towers

E2/5. Két markoló is három gödröt ás.
   two excavator three hole-ACCUSATIVE digs
   'Two excavators are digging three holes.'
Left picture: 3 holes, 6 excavators; Right picture: 2 excavators, 6 holes

E2/13. Három gödröt is két markoló ás.
   three hole-ACCUSATIVE two excavator-NOMINATIVE digs
'Three holes (each) are being dug by two excavators.' (Literally: 'Three holes each, two excavators are digging.')
Left picture: 3 holes, 6 excavators; Right picture: 2 excavators, 6 holes

E2/10. Két cica is három párnán alszik.
two cat three pillow-on sleeps
'Two cats (each) are sleeping on three pillows.'
Left picture: 3 pillows, 6 cats; Right picture: 2 cats, 6 pillows

E2/20. Három párnán is két cica alszik.
three pillow-on two cat sleeps
'On three pillows (each), two cats are sleeping.
Left picture: 3 pillows, 6 cats; Right picture: 2 cats, 6 pillows

E2/15. Két széken is három esernyő van.
two chair-on three umbrella is
'On two chairs (each), there are three umbrellas.'
Left picture: 2 chairs, 6 umbrellas; Right picture: 3 umbrellas, 6 chairs
E2/8. Három esernyő is két széken van rajta.
three umbrella two chair-on is on
'Three umbrellas (each) are (placed) on two chairs.'
Left picture: 2 chairs, 6 umbrellas; Right picture: 3 umbrellas, 6 chairs

In the eight test sentences, the two quantified expressions represented four different pairs of thematic roles: agent – patient; (inanimate) actor – patient; agent – location; location – patient. Each pair of quantifiers was presented in both linear orders. That is, the conditions differed in the following respects: (AG stands for agent, AC for actor, PAT for patient, LOC for location.)

1. AGq PATq V Condition (E2/18): Agent first, Patient second
2. PATq AGq V Condition (E2/3): Patient first, Agent second
3. ACq PATq V Condition (E2/5): Actor first, Patient second
4. PATq ACq V Condition (E2/13): Patient first, Actor second
5. AGq LOCq V Condition (E2/10): Agent first, Location second
6. LOCq AGq V Condition (E2/20): Location first, Agent second
7. LOCq PATq V Condition (E2/15): Location first, Patient second
8. PATq LOCq V Condition (E2/8): Patient first, Location second

The test question in each of these conditions was whether the child associated the doubly quantified sentence with a drawing representing its direct scope reading, or with a drawing representing its inverse scope reading.

6.2. Results
There was no difference between the Right-hand side Direct/Inverse responses (59%, 61%) and the Left-hand side Direct/Inverse responses (61%, 39%) $X^2(df=1)=2.94$, $p=0.09$.
The males’ and females’ Direct and Inverse responses (Directmale=23%, Directfemale=20%, Inversemale=31%, Inversefemale=27%) were not different, either ($X^2(df=1)=0.02$, $p=0.88$). So only the analyses of the test sentence–picture pairs are presented below.

1. AGq PATq V Condition (E2/18)
   Direct scope (AGq > Pq): 78%.
   Inverse scope (AGq < Pq): 22%.
2. PATq AGq V Condition (E2/3)
   Direct scope (PATq > AGq): 58%.
   Inverse scope (PATq < AGq): 42%. (1 subject answered: I don’t know.)
3. ACq PATq V Condition (E2/5)
   Direct scope (ACq > PATq): 85%.
   Inverse scope (ACq < PATq): 15%.
4. PATq ACq V Condition (E2/13)
   Direct scope (PATq > ACq): 22%
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Inverse scope (PAT_q < AC_q): 78%

5. AG_q LOC_q V Condition (E2/10)
   Direct scope (AG_q > LOC_q): 80%
   Inverse scope (AG_q < LOC_q): 20%

6. LOC_q AG_q V Condition (E2/20)
   Direct scope (LOC_q > AG_q): 32%
   Inverse scope (LOC_q < AG_q): 68%

7. LOC_q PAT_q V Condition (E2/15)
   Direct scope (LOC_q > PAT_q): 76%
   Inverse scope (LOC_q < PAT_q): 24%

8. PAT_q LOC_q V Condition (E2/8)
   Direct scope (PAT_q > LOC_q): 22%
   Inverse scope (PAT_q < LOC_q): 78%

Figure 2  The ratio between the Direct and Inverse responses for the test sentences in Exp 2.

6.3. Discussion
We expected that the children’s preferred scope order would be determined by either the linear order or the thematic ranking of the two quantifiers.
   Those relying on linear order would choose the pictures representing the direct scope readings of test sentences (with the first quantifier having scope over the second one). This is the strategy that our control group of university students followed. We administered Experiment 2 to 44 students at Budapest University of Technology and Economics, and we received the following results:

Results of the adult control group:
1. AG_q PAT_q V Condition (E2/18)
   Direct scope (AG_q > PAT_q): 98%.
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Inverse scope \( (AG_q < P_q): 2\% \).

2. \( PAT_q AG_q V \) Condition (E2/3)
   - Direct scope \( (PAT_q > AG_q): 86\% \).
   - Inverse scope \( (PAT_q < AG_q): 14\% \).

3. \( AC_q PAT_q V \) Condition (E2/5)
   - Direct scope \( (AC_q > PAT_q): 91\% \).
   - Inverse scope \( (AC_q < PAT_q): 9\% \).

4. \( PAT_q AC_q V \) Condition (E2/13)
   - Direct scope \( (PAT_q > AC_q): 84\% \).
   - Inverse scope \( (PAT_q < AC_q): 16\% \).

5. \( AG_q LOC_q V \) Condition (E2/10)
   - Direct scope \( (AG_q > LOC_q): 98\% \).
   - Inverse scope \( (AG_q < LOC_q): 2\% \).

6. \( LOC_q AG_q V \) Condition (E2/20)
   - Direct scope \( (LOC_q > AG_q): 86\% \).
   - Inverse scope \( (LOC_q < AG_q): 14\% \).

7. \( LOC_q PAT_q V \) Condition (E2/15)
   - Direct scope \( (LOC_q > PAT_q): 100\% \).
   - Inverse scope \( (LOC_q < PAT_q): 0\% \).

8. \( PAT_q LOC_q V \) Condition (E2/8)
   - Direct scope \( (PAT_q > LOC_q): 80\% \).
   - Inverse scope \( (PAT_q < LOC_q): 20\% \).

The scope interpretation strategy relying on thematic ranking was expected to be based on the hierarchy of thematic roles in (xv) (this is the hierarchy that also determined the scope interpretation of Chinese children tested by Lee (2003)).

(xv) Thematic hierarchy:
   - actor > location > patient

The scope interpretation strategy relying on thematic ranking would choose the direct scope reading in Conditions 1, 3, 5, and 7, and the inverse scope reading in Conditions 2, 4, 6, and 8. In Conditions 1, 3, 5, and 7, where the first quantifier was also thematically more prominent, the great majority of the children (76-85\%), indeed, chose the picture representing the direct scope interpretation. This is as expected, as in this case both the strategy of assigning wide scope on the basis of linear order, and the strategy of assigning wide scope on the basis of thematic prominence converged on the initial quantifier.

In Conditions 2, 4, 6, and 8, where the first quantifier was thematically less prominent than the second one, we expected some subjects to assign scope on the basis of linear precedence, and others to assign scope on the basis of thematic prominence. In Condition 2 (\( PAT_q AG_q V \)), involving a patient quantifier followed by an agent quantifier, slightly more subjects (58\%) assigned wide scope to the initial patient than to the thematically more prominent agent (42\%). In the other three conditions, on the other hand, the great majority (78\%, 68\%, and 78\%) assigned wide scope to the thematically more prominent second quantifier. It is not obvious why the distribution of the two strategies was more even in the 2nd, \( PAT_q AG_q V \).
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Condition than in the 4th (PATq ACq V), 6th (LOCq AGq V), and 8th (PATq LOCq V) Conditions. We hypothesize that this difference may have derived from the different grouping of objects in the visual representations. In the 2nd, PATq AGq V Condition both pictures represented the two types of objects (boys and towers) intermingled. In Conditions 4, 6, and 8, on the other hand, one of the pictures showed one type of objects conspicuously grouped into two identical sets (cf. the holes in the right picture in Condition 4, the pillows in the right picture of Condition 6, and the umbrellas in the left picture in Condition 8). Children tended to identify objects grouped into such conspicuous identical sets as the multiplicand, i.e., the narrow scope expression – whatever its linear order and its thematic role. If this observation is right, then the visual grouping of objects might have contributed to the strong preference for direct scope in Conditions 3, 5, and 7, as well.

This assumption would also explain why sentence E1/6 (Három maci is két autóval játszik 'Three teddy bears (each) are playing with two cars') of Experiment 1 was accepted as a true statement about a picture showing its inverse scope interpretation, involving six teddy bears and two cars, by 63% of the children. In this S_q > O_q Condition the quantified object assigned wide scope was neither initial, nor more prominent either functionally or thematically than the initial quantifier. The initial agent-subject could presumably be assigned narrow scope by so many children because it was visually represented by two identical sets (triplets) of teddy bears. 19 children of those who accepted this sentence under the inverse scope reading also participated in Experiment 2. Their direct scope answers in conditions 3,5,7, and their inverse scope answers in condition 4, 6, and 8 display the correlation r_{Pearson}=0.85, p< 0.001. This fact confirms that their judgement of sentence E1/6 and their selection of scope order in Experiment 2 were not random but were manifestations of interpretation strategies.

Nevertheless, the thematic hierarchy in (xv) must also have played a role in determining children’s scope interpretation – as the 42% of subjects who chose the picture representing the inverse scope reading in Condition 2 (PATq AGq V) had no other obvious motivation for assigning wide scope to the second quantifier than its thematic prominence.

In sum: whereas in adult Hungarian grammar the scope order of two preverbal quantifiers is determined by their linear order, the majority of the children tested in Experiment 2 assigned wide scope to the initial quantifier only if it was also thematically more prominent then the second quantifier. Narrow scope assignment to a quantifier (i.e., its interpretation as the multiplicand) appears to have been facilitated if the of objects denoted by it were grouped into conspicuous identical sets in the visual representation. Apparently, children working with analog magnitude representations of approximate numerosities rely more heavily on visual resources than adults with access to exact arithmetic. The thematic hierarchy in (xv) must also have played a role in determining children’s scope interpretation – as the 42% of subjects who chose the picture representing the inverse scope reading in Condition 2 (PATq AGq V) had no other obvious motivation for assigning wide scope to the second quantifier than its thematic prominence.

7. Experiment 3: Acting out the primary interpretation

Experiment 1 has shown that doubly quantified sentences involving the particle is also have a non-distributive (collective) reading for children, in addition to their two multiplicative interpretations. For adults, this reading is a highly marked option requiring contextual support (recall that the adult control group assigned to sentence E1/15 paired with a picture showing its collective reading the median rating 2). For the children participating in Experiment 1, however, this reading was freely accessible; 93% of the children accepted it (whereas the distributive interpretation was accessible to 91% of them).

Experiment 3 had two purposes. On the one hand, we wanted to find out whether the distributive or the collective reading of doubly quantified sentences is primary for
preschoolers, and what their choice of primary reading depends on. We tested the role of two factors. Experiments 1 and 2 demonstrated that most children converge on a distributive direct scope reading if the linear order and the thematic order of the two quantifiers coincide. Relying on this observation we hypothesized that the elicitation of the distributive interpretation as opposed to the collective reading might be facilitated if the linear order and the thematic ranking of the quantifiers correspond to each other. We also suspected that the distributive versus collective reading of a doubly quantified sentence is influenced by the pragmatics of the event. Thus a sentence like *Three boys ate two apples* is likely to evoke the distributive reading, whereas in the case of *Three boys watched two dancers*, the collective reading comes first to mind.

On the other hand, we also wanted to test whether children merely recognize the product of a linguistically encoded multiplication, or they can also compute the product themselves. Therefore, in the third experiment we asked children to act out the meanings of test sentences they heard, using puppets and toys.

**7.1. Method**

**Subjects:**
48 subjects, 25 (52%) males and 23 females (48%) participated in the experiment. They were recruited from the same kindergartens as in Experiments 1 and 2. The mean age of the subjects was $M=6;6$, $SD=4$ months. The mean age for males was $M=6;7$, $SD=4$ months, for females was $M=6;5$, $SD=4$ months ($F(1/39)=4,24, p<0,05$). The ages of the children by kindergarten: $EK_{n=17}: M=6;7$, $SD=4$ months, $HK_{n=17}: M=6;4$, $SD=3$ months, $TK_{n=14}: M=6;7$, $SD=5$ months. With respect to the ages of the subjects there was no difference between the kindergartens ($F(2/38)=2,51, p=0,09$).

**Procedure:**
The child, the experimenter, and a helper were seated at a table in a quiet room of the kindergarten. The helper had the hedgehog puppet on her hand. The child had a 25 cm x 20 cm mat in front of her/him, surrounded by arrays of 6 identical little bears, 6 identical little cars, 6 identical little boats, 6 identical candies, and two little benches. The experimenter explained that that was a kindergarten for little bears. The bears like to play on the mat, and their favorite toys are little cars and little boats. When they are tired, they sit down on the benches. Good little bears receive candies. The child and the hedgehog were going to play a game; the hedgehog would tell the child what she would like to see on the mat, and the child should set up the situation.

The child had to set up 12 situations, among them 5 test cases and 7 fillers. Each one started with the experimenter asking the hedgehog what she would like to see on the mat. The hedgehog uttered a sentence, which the child acted out with the toys on the table. When ready, (s)he received some positive feedback („that’s great”, „well done”); then (s)he was asked to move the bears and toys back to their original places. The experimenter recorded whether the scenes set up represented the distributive or the collective readings of the test sentences.

The four test sentences were of the type tested in the first two experiments:

E3/6. Három maci is két hajóval játszik.
three teddy bear two boat-with plays
'Three teddy bears (each) are playing with two boats.'

two car-with three teddy bear plays
'With two cars (each), three teddy bears are playing.'

E3/12. Három maci is két cukorkát kapott.
three teddy bear two candy-ACCUSATIVE received
'Three teddy bears (each) received two candies.'

two bench-on three teddy bear sits
'On two benches (each), three teddy bears are sitting.'

The following sentence was first intended to represent a filler, but as it was relevant from the point of view of our research question, it was eventually included among the test cases to be analyzed. It involves a numerically modified dative argument and an indefinite subject represented by a bare nominal. It contains no distributive is, but the meaning of the verb supports the distributive reading also without it:

four teddy bear-DATIVE is car-POSSESSIVE
'Four teddy bears have a car.'

Sentences E3/6 and E3/3 are minimal pairs differing in the relative order of the quantifiers. Sentences E3/12 and E3/9 differ from them in the type of relation denoted by their verbs. Játsszik 'play' in E3/3 and E3/6 denotes an activity which can involve the players and the toys either collectively or distributively (the players can play together or individually, and the toys can be shared or can be assigned to individual players). We assumed that the relation established by the verb kap 'receive' between the receivers and the objects received in E3/12 would be more likely to be distributive; different receivers can easily be assigned different set of objects (although receiving objects collectively is also conceivable). Ül 'sit' expresses an even more obviously distributive relation between locations and agents (the most likely scenario involving a set of benches and a set of sitting persons is such that different benches host different sets of persons). The possessive predicate in E3/11 also denotes a primarily distributive relation between the possessors and the possessum. That is, these sentences instantiated the following conditions:

1. S_q O_q [V_{coll/dist}] Condition (E3/3)
2. O_q S_q [V_{coll/dist}] Condition (E3/6)
3. S_q O_q [V_{dist, coll}] Condition (E3/12)
4. LOC_q S_q [V_{dist (coll)}] Condition (E3/9)
5. DAT [V_{dist (coll)}] S Condition (E3/11)

The test question was how subjects interpret doubly quantified sentences containing the particle is in these conditions; whether their choice between the distributive and the collective interpretations is affected by the linear order of the two quantifiers, and the pragmatics of the verb.

7.2. Results
The difference between the mean numbers of Collective and Distributive set-ups of the test sentences by the subjects (M_{collective}=3,10 (SD=0,69), M_{distributive}=1,77 (SD=0,90)) was significant (F(1/94)=65,79, p<0,001). It was also significant within the males and the females F_{male}(1/48)=33,75, p<0,001; F_{female}(1/35)=30,84, p<0,001. However, there was no difference
between the sexes in the number of Collective and Distributive responses
\(F_{\text{collective}}(1/46)=0.06, p=0.80; F_{\text{distributive}}(1/46)=0.16, p=0.68\). The mean numbers of Collective
responses given by the children of the three kindergartens were not different, either
\(F(2/45)=0.36, p=0.69\).

The overall number of collective set-ups was 154 (65%), the overall number of distributive
set-ups was 85 (35%). The overall number of the set-ups by males and females was 124
(52%) and 115 (48%), respectively. The differences between the numbers of males and
females and the numbers of response types given to the test sentences (Collective\textsubscript{male-female}= 52%, 48%, Distributive\textsubscript{male-female}=52%, 48%) were not significant \((X^2(\text{df}=1)=0.00, p=0.98)\).

In Conditions 1 and 2, every child acted out the collective reading of the sentence. In
Condition 3, 25% of the subjects chose the distributive interpretation. That is, the majority of
children set up a scene where a group of three bears received two candies between them. In
condition 4, the distributive interpretation prevailed; 65% of the subjects set up a scene with 3
bears sitting on each of two benches. 35% of the subjects joined the two benches and placed
the three bears across them.

1. \text{S\_q O\_q [V\text{coll/dist}] Condition (E3/3)}
   Distributive interpretation: 0%
   Collective interpretation: 100%

2. \text{O\_q S\_q [V\text{coll/dist}] Condition (E3/6)}
   Distributive interpretation: 0%
   Collective interpretation: 100%

3. \text{S\_q O\_q [V\text{dist (coll)}] Condition (E3/12)}
   Distributive interpretation: 25%
   Collective interpretation: 75%

4. \text{LOC\_q S\_q [V\text{dist}] Condition (E3/9)}
   Distributive interpretation: 65%
   Collective interpretation: 35%

5. \text{DAT [V\text{dist (coll)}] S Condition (E3/11)}
   Distributive interpretation: 90%
   Collective interpretation: 10%
7.3. Discussion
As was shown by experiment 1, children can associate doubly quantified sentences both with a distributive/multiplicative and a collective reading. Experiment 3 aimed to examine which of the two readings is primary for them, which reading they choose when hearing a doubly quantified sentence out of context. Our tests examined the effect of two types of conditions: the correlation between the linear order and the thematic ranking of the two quantifiers, and the type of pragmatic relation established between them by the verb. The former condition proved to have no effect. The fact that in Condition 1 the linear order of the two quantifiers matched their thematic ranking did not help elicit the distributive/multiplicative reading; every child chose the collective interpretation both in Condition 1, and in Condition 2, representing the opposite order of quantifiers.

The selection of the preferred reading proved to be determined by the pragmatics of the verb of the sentence (the lack of context excluded the influence of other pragmatic factors). The verbs used in the test sentences denoted situations establishing a distributive – one-to-many – relation between the participants with different degrees of likelihood. The proportion of distributive responses corresponded to the likelihood of the pragmatic distributivity of the given event. Thus sentences involving teddy bears playing with toys evoked the collective interpretation in every case. The sentence involving teddy bears receiving candies elicited the distributive reading in 25% of cases (kindergarteners are apparently taught to share candies). The sentence involving benches and teddy bears sitting on them was interpreted distributively by the majority (65%) of the subjects. The sentence Négy macinak van autója ‘Four teddy bears have a car’ was interpreted distributively by 90% of the children, even though this sentence contains no distributive is particle, and the word order Q1 V Q2 does not enforce the distributive reading, either. It must have been the pragmatics of the possessive relation that elicited the distributive interpretation for most children.

In sum: the results of Experiment 3 have confirmed the hypothesis that preschoolers interpreting doubly quantified sentences have the means of computing both their collective reading and their distributive interpretation. Whereas experiments 1 and 2 have only shown that preschoolers can recognize a visually represented situation as the product of a linguistically encoded multiplication, Experiment 3 has proved that they can also actively...
calculate multiplication, computing the product by themselves. At the same time, they opted for the distributive interpretation of a doubly quantified sentence only if the pragmatic conditions made the collective reading implausible. Apparently, the default reading for preschoolers is the collective reading, presumably because its computation imposes a lesser load on their cognitive system. The distributive function of the particle *is* attached to numerically modified expressions must be fixed at a later age.

8. General discussion
This research was motivated by the hypothesis put forth in developmental literature that the approximate number system that infants and preschoolers have access to should support not only addition and subtraction, but multiplication, as well. We have assumed that – visually induced multiplication being often indistinguishable from repeated addition – the evidence for intuitive multiplication should be sought for in language, e.g., in certain types of sentences containing two numerical quantifiers. When interpreting such sentences, the listener – optionally or obligatorily, depending on linguistic clues – treats the two quantifiers as a multiplier and a multiplicand, and computes the product of multiplication. This process is presumably based on the approximate number system. Not only preschoolers calculate the product of multiplication relying on approximate magnitudes but also adults do so when the sentence contains numerical quantifiers whose product they have not memorized in the multiplication table.

Our experiments have shown that Hungarian preschoolers with no linguistic training can interpret doubly quantified sentences correctly, carrying out the multiplication encoded by syntactic means. They can recognize the visually shown product of a linguistically encoded multiplication, and they can also compute the product actively by setting up a situation representing it. This confirms the hypothesis that multiplication is part of the biologically determined arithmetic toolkit of humans.

At the same time, children’s strategies of interpreting doubly quantified sentences appear to be more flexible than those of adults. In Hungarian adults’ grammar, linguistic clues indicate if a sentence is to be interpreted distributively/multiplicatively or collectively. Preschoolers tend to ignore these clues; they assign a multiplicative/distributive reading to a doubly quantified sentence only when pragmatics makes the collective reading implausible. They treat the collective interpretation of doubly quantified sentences as default presumably because its computation imposes a lesser load on their cognitive system.

The fact that children’s grammar of quantification is more flexible, containing more interpretative possibilities, than adults’ grammar has also been observed in Chinese by Zhou and Crain (2009). Obviously, children everywhere start out with the same options provided by Universal Grammar, and acquire the language-specific constraints of their mother tongue gradually, at a later age.

Children are much more flexible also in determining scope order, i.e., in selecting the multiplier and the multiplicand. Whereas in Hungarian adults’ grammar scope order depends on the linear order of quantifiers, children have been found to follow at least three strategies. They may assign wide scope to the initial quantifier, as adults do, or to the quantifier that is thematically more prominent. A third factor affecting scope order for children may be the visual representation/grouping of the sets denoted by the two quantifiers, indicating that children’s intuitive arithmetic, based on analog magnitude representations, draws on both linguistic and visual resources.
The linguistic roots of multiplication

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Appendix
Experiment 1

E1/3. Two towers are being built by three boys.

E1/6. With two cars, three teddy bears are playing.

E1/9. Three boys are building two towers.
E1/12. Three teddy bears are playing with two cars.

E1/15. Two teddy bears have three cars.
Experiment 2

E2/3. Three towers are being built by two boys.

E2/5. Two excavators are digging three holes.

E2/8. Three umbrellas are on two chairs.
E2/10. Two cats are sleeping on three pillows.

E2/13. Three holes are being dug by two excavators.

E2/15. On two chairs, there are three umbrellas.
E2/18. Two boys are building three towers.

E2/20. On three pillows, two cats are sleeping
Experiment 3

E3/3. With two cars, three teddy bears are playing.
E3/6. Three teddy bears are playing with two boats.
E3/9. On two benches, three teddy bears are sitting.
E3/11. Four teddy bears have a car.
E3/12. Three teddy bears received two candies.