Obligations, permissions and transgressions: An alternative approach to deontic reasoning

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Abstract

This paper proposes a logic of transgressions for obligations and permissions. A key objective of this logic is to allow deontic conflicts (Lemmon 1962) but without appealing to defeasible or paraconsistent reasoning, or multiple levels of obligation. This logic of transgressions can be viewed as conceptually related to those approaches that formulate obligations in terms of “escaping” from a sanction (Prior 1958; Nowell-Smith & Lemmon 1960), and its modal variants (Anderson 1958; Kanger 1971), but where the notion of a transgression is more fine-grained than a single “sanction”.

1 Introduction

Deontic statements are used to describe or convey obligations and permissions, such as:

(1) a. You should close the window.
   b. You must pay your taxes.
   c. You may go to the beach.
   d. You are allowed to walk on the grass.

The paradigm treatment for reasoning with obligations and permissions is Standard Deontic Logic (SDL), whose genesis is usually attributed to von Wright (1951). In SDL, obligations and permissions are treated as modal statements of the form $O(a)$ and $P(a)$, where $O$ and $P$ represent operators for obligations and permissions, respectively, and $a$ represents that which is obligatory or permitted. In SDL, $a$ is assumed to be a proposition. SDL adopts a number of axioms and inference rules which effectively extend classical logic so that it can be used to reason with such statements. The relevant behaviour can be modelled using possible worlds, and some notion of an “ideal world”. In brief, something is obligatory if it takes us “nearer” to such an ideal world, and is permitted if it is compatible with an ideal world.

Unfortunately, there are many cases where SDL does not appear to support our intuitive understanding of the meaning of natural language statements about permissions and obligations. One such case considered here is that of conflicting obligations, which SDL cannot handle. Here, we will consider an alternative approach to reasoning with deontic expressions, and see how it applies in such cases. We will focus on a straightforward reparational obligations:

(2) a. You ought not steal.
   b. If you steal, you ought to make amends.

and “Plato’s Dilemma” (Lemmon 1962):¹

¹There are other well-known examples — such as the Good Samaritan paradox (Prior 1958), and Chisholm’s Contrary-to-duty paradox (Chisholm 1963) — the analysis of which may involve conflicts in
When obligations and permissions are modelled in terms of possible worlds, in the usual way — where what is obligatory is what holds in some ideal world, or which would take us nearer to an ideal world — such cases appear to require various complications involving sub-optimal worlds (see Jones & Pörn 1985 for example), or the adoption of non-standard logical notions, such as paraconsistency (Costa & Carnielli 1986) and non-monotonicity (Bonevac 1998), or the use of multiple levels of obligation (Åqvist 1967).

Here we propose a different approach which allows us to reason in the face of unmet obligations without resorting to notions of sub-optimality, paraconsistency, or non-monotonicity in the semantic theory. In some sense it can be thought of as a variant of input/output logic (Makinson & van der Torre 2003), except that the “output” (what can be concluded) is not an indication of what is obligatory given some state of affairs, but is instead an indication of whether the “input” has transgressed the system of rules associated with some authority. In spirit, this analysis is similar to the approaches of Prior (1958); Nowell-Smith & Lemmon (1960); Anderson (1958), but with a more fine-grained notion of what counts as a “sanction”. An obedient subject may view their goal as being to minimise the number of transgressions that they make, with respect to the relevant authorities. This gives us scope to reason with obligations even if a code of conduct is inconsistent with itself, or inconsistent with other authorities. We also show how permissions can be formulated in such a framework, and give a sketch of entailment-like properties for rational and coherent authorities.

2 The Theory

Here the basic notions of the theory are presented, followed by rules governing obligations, permissions, and their combination. The representation of conditional obligations is also briefly discussed.

2.1 Basic Notions

Assume we have some system of obligations $O_A$ imposed by authority $A$. If an obligation for $a$ to be the case is part of $O_A$, then we write $O_A(a)$. In the event that an obligation $O_A(a)$ is not satisfied, then a transgression has occurred, which we write $T_A(a)$. Here we use $\Gamma, O_A \vdash T_A(a)$ to denote that a transgression of an obligation of $O_A$ for $a$ to be the case follows in context (state of affairs) $\Gamma$. We assume that $\Gamma$ is a classical theory that is free from statements concerning obligations and permissions.

The unfulfillability of a system of obligations $O_A$ (for any context $\Gamma$), is to be expressed by $O_A \vdash \bot_O_A$, where $\bot_O_A$ is intended to indicate the inconsistency of the obligations $O_A$ of $A$. Indexing the expression $\bot$ with the source of the inconsistency allows us to avoid conflating different sources of inconsistency. In particular, the inconsistency of a system of obligations (and permissions) should not lead to an inconsistency in the logical theory itself. Similar notation is used for inconsistent permissions, and inconsistencies between obligations and permissions for a given authority.

what obligations are entailed. These are not discussed here, partly because they contain subtle issues that may obscure the focus of this paper. Some problems relating to the Good Samaritan are discussed in Fox (2009).

Kanger (1971) gives an account that is closely related to that of Anderson (1958) except it uses a notion of obligation fulfillment in place of a sanction.
The permissions $P_A$ of an authority $A$ could be thought of as characterising things $a$ that should not give rise to transgressions according to $A$. If $a$ is permitted by $A$, we can write $P_A(a)$.

Using these notions, we can define aspects of the behaviour of obligations, permissions and transgressions. Here we are assuming what might be characterised as a “proof-theoretic” approach, where behaviours and definitions are given directly in terms of the language of obligations, permissions and transgressions, rather than by way of some model theory. To avoid a potential source of additional complexity, we assume that we are dealing with descriptions of obligations and permissions, rather than statements that in and of themselves impose obligations or grant permissions. We also do not seek to distinguish between obligations to do versus obligations to be in this account.

The expressions (i) $\perp_{O_A}$, (ii) $\perp_{P_A}$, and (iii) $\perp_{O_A,P_A}$ shall be used to denote an inconsistency in the system of (i) obligations, (ii) permissions, or (iii) both, respectively, of authority $A$ in the following rules for systems of obligations $O_A$ and permissions $P_A$.

### 2.2 Obligations

If $A$ imposes the obligation that $a$, and $\Gamma$ supports the negation of $a$, then from $\Gamma$ it follows that there is a transgression of the requirement for $a$ to be the case.

**Rule 1 (Transgression)** If $O_A(a)$ and $\Gamma \vdash \neg a$, then $\Gamma, O_A \vdash T_A(a)$.

We could also consider negated obligations, but will refrain from doing so. We wish to remain uncommitted as to whether a negated obligation to $a$ necessarily implies the granting of a permission to $\neg a$.

**Rule 2 (Fulfillability/Coherence)** If for all $\Gamma$ it is the case that $\Gamma, O_A \vdash T_A(b)$ for some $b$, then the obligations $O_A$ of $A$ are unfulfillable (and $A$ can be described as imposing inconsistent or incoherent obligations), for which we write $O_A \vdash \perp_{O_A}$. Otherwise they are fulfillable (and the obligations imposed by $A$ can be described as consistent or coherent).

Unfulfillability describes the case where all the obligations of an authority cannot be fulfilled as there is no way that they can all be true without leading to an inconsistency in the representation of the state of affairs.

**Rule 3 (Vacuity)** If for $\Gamma \nvdash \perp$, $\Gamma, O_A \nvdash T_A(b)$, for all $b$, then the obligations $O_A$ are vacuous, that is, the content of the obligations are tautologies (for which we could write $O_A \vdash \top_{O_A}$).

Vacuity describes the case where all of an authority’s obligations are trivially satisfied because they are universally valid.

This transgression-based account adopts a neutral moral perspective. In particular, the account does not aim to capture any moral imperative to comply with obligations. Rather, it seeks only to support reasoning about whether a transgression may be deemed to have occurred according to some authority, regardless of whether a given subject feels bound to comply with the obligations of that authority. Subjects may not even be aware of the existence of such obligations, or authorities. Even so, the intuitive validity of the

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3 The view adopted here is that the primary role of model theory is in demonstrating consistency, rather than explicating behaviour, a general approach advocated in Fox (2000) and elsewhere.

4 Hansen et al. (2007) discuss the notion of coherence, among other things, and how it is best defined.
approach can be assessed by considering whether an agent who seeks to avoid (additional) transgressions would be deemed to be complying with the relevant, coherent and non-vacuous obligations. An alternative test would be to consider the case of a third party coming to a judgement about whether an authority’s rules have been breached. In this setting, any moral or pragmatic imperative to comply with an obligation can be considered as a meta-level obligation for the subject to avoid the transgression(s) in question (perhaps because the transgression carries a cost for the agent).

A couple of details are set aside in this account. The first concerns when the existence of a transgression should be determined; there is no attempt here to consider the time that may be allowed before compliance is evaluated, or hypothetical evaluations of possible future transgressions. This is something that would need to be considered in the event that the theory were used as a guide to moral or legal behaviour.

The second detail concerns who is deemed to be responsible for a given transgression. We might normally assume that a in O_A(a) is some agentive statement whose subject indicates the responsible entity, but it is possible to contemplate cases where some other entity would bear the penalty for a failure to comply with an obligation, or where the obligation makes no explicit mention of an intended subject. The current proposal says nothing on this topic.

2.3 Permissions

Now let us consider permissions.

Rule 4 (Consistent/Inconsistent) If P_A(a) and P_A(b), but a ⊢ ¬ b, then P_A is inconsistent, for which we can write P_A ⊢ ⊥_{P_A}. Otherwise it is consistent, for which we write P_A ⊬ ⊥_{P_A}.

This captures the case where an authority is (in)consistent in what it permits. We wish to avoid what might seem a more obvious definition, that from P_A(b) we can infer Γ, P_A ⊬ T_A(b). This is because such a definition would make the notion of entailment itself (⊢) inconsistent in the face of an inconsistent system of authority, something we are trying to avoid.

2.4 Obligations and Permissions

A given authority may seek to impose obligations and grant permissions. What we call mutually consistent obligations and permissions are ones where no potential transgression of the obligations contradicts the permissions that have been granted.

Rule 5 (Mutually Consistent/Inconsistent) If for all Γ such that Γ ⊬ ⊥ (i.e. consistent Γ), and all a such that P_A(a), there is no b such that ⊢ b → a and Γ, O_A ⊢ T_A(b), then O and P are mutually consistent. Otherwise they are inconsistent, for which we write O_A, P_A ⊬ ⊥_{O_A, P_A}.

Other rules and definitions characterising an authority’s system of obligations and permissions are possible. Those presented here are offered as examples to illustrate the general objectives of this approach. An obvious simplification would be to have only one notion of deontic inconsistency for an authority, or allow a general notion of deontic inconsistency to follow from any of ⊥_{O_A}, ⊥_{P_A}, ⊥_{O_A, P_A}. We might also seek to collapse O_A and P_A into a single notion. One case that has not been considered is when the two authorities
have mutually conflicting obligations or permissions. It should be fairly clear how such an extension can be incorporated.

It is important to note that this initial sketch is deliberately weak; although transgressions are being used to characterise properties of authorities, no indication has been given about when one transgression implies another. Any such elaborations have to be considered with great care, especially if we wish to allow inconsistent systems of authority without accidentally introducing inconsistencies in the theory used to describe their behaviour. In effect, what is proposed could encompass a range of more specific theories.

2.5 Conditionals

In the literature on deontic reasoning, much is made of the difficulty of analysing conditional obligations (McNamara 2006), which has motivated the use of dyadic deontic modalities (van Fraassen 1972, for example). Although more could be said, here it will merely be noted that conditional obligations can be expressed in the form \( p \rightarrow O_A(a) \), meaning that the obligation \( O_A(a) \) applies when \( p \) is true. It may be preferable for the notation to indicate that the conditionality is part of the authority \( A \)’s code, which would be better captured by \( O_A(p \rightarrow a) \). It can be argued that this should still be analysed in terms of propositional detachment (the antecedent being \( p \)), rather than deontic detachment (the antecedent being \( O_A(p) \)), following Castañeda (1981).

3 Conflicting Obligations

This analysis of deontic statements is able to cope with both reparational and conflicting obligations.

3.1 Reparational Obligations

Let us consider the reparational example (2), repeated here.

(2) a. You ought not steal.
   b. If you steal, you ought to make amends.

If you steal, then a transgression will have occurred. If you then fail to make amends, a second transgression will occur. There is no need to appeal to powerful non-standard notions of inference, or face the question of determining some consistent fixed order of priority for different obligations.

As noted in the introduction, the idea of transgressions is in some respects similar to the characterisation of an obligation as something that will give rise to a “sanction” if it is unsatisfied (Prior 1958; Anderson 1958). A key difference is that we do not just have one sanction. This is what allows us to reason with reparational obligations. Nowell-Smith & Lemmon (1960) tried to distinguish between different sanctions by considering agency, but that approach would not work in this case as both obligations apply to the same subject.

3.2 Plato’s Dilemma

Given that the proposed analysis is intended to allow inconsistencies in systems of obligations, it is appropriate to consider “Plato’s Dilemma” where we have an indirect conflict between obligations, but where we would normally say that one obligation has priority over the other (Lemmon 1962). The example (3) is repeated here.
a. I’m obligated to meet you for a light lunch meeting at the restaurant.
b. I’m obligated to rush my choking child to the hospital.

Again, when considered in terms of transgressions we do not need to appeal to any notion of defeasibility. If it is not possible to fulfill both obligations, then a transgression will arise, but there will not necessarily be an inconsistency in the theory used to represent and reason with the obligations.

In determining what an agent should do in the face of such a conflict, we might be tempted to consider introducing some form of ordering. But this need not be on the obligations themselves. In deliberating about which obligation to fulfill and which to transgress, an agent may consider both the source of authority, and cost of the transgression that will arise. This may be no different from the general case of an agent considering the costs and benefits of alternative outcomes of their plans and actions.

4 Deontic Reasoning with Transgressions

It is appropriate to consider the nature of entailments that are or should be supported by a deontic system expressed in terms of a logic of transgressions. In the current formulation, this would include entailments and associated behaviours for obligations, permissions, and transgressions, and between expressions involving both obligations and permissions.

Here, comparisons will be made between this system and SDL, concentrating on the nature of the entailments that are supported, and the relationship between statements involving permissions and obligations. In general, it seems appropriate to determine what analogues there may be for the axioms, rules and theorems of SDL. We will consider two theorems of SDL.

(4) If $\vdash a \rightarrow b$ then $\vdash Oa \rightarrow Ob$.

(5) $O(a) \rightarrow P(a)$

Theorem (4) supports the argument that $O(a)$ follows from $O(a \wedge b)$, for example. Theorem (5) captures the view that if you are obliged to do something, you must also be permitted to do it.

In the proposed system, one thing to consider is the behaviour of transgressions, as in (6), which is entailed by Rule 1.

(6) If $O_A(a)$ and $O_A(b)$ and $\vdash a \rightarrow b$, then $O_A \vdash T_A(a) \rightarrow T_A(b)$.

This can be considered somewhat akin to (4). This inference rule is conditional on both $a$ and $b$ already being obligations. It avoids unconditional inferences concerning transgressions of obligations that have not been stated, even if such obligations can be derived in the case of SDL. Without such constraints, an authority would implicitly condone obligations that have not been given explicitly. From this alone it is apparent that the current proposal is weaker than SDL. Unlike SDL, it does not require obligations such as $O_A(a)$ to follow from other obligations, such as $O_A(a \wedge b)$. This can be justified by considering the example You are obliged to jump out of the window and land on the pile of mattresses (Jackson 1985). We might be reluctant to conclude that there is a distinct and unqualified

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5 Specifically, we do not need to follow Åqvist (1967) and introduce a family of ordered modal operators.

6 The proposal to distinguish the essentially deontic component from other aspects of reasoning is not new. For example, Prakken (1996) discusses a similar idea in the case of deontic reasoning and defeasibility.
obligation You are obliged to jump out of the window, although a failure to “jump out of the window” would imply a failure to “jump out of the window and land on the pile of mattresses”.

Other aspects of behaviour may be characterised by rules or theorems that indicate cases where an authority may be judged to be inconsistent, as with (7), which is entailed by Rule 5.

(7) If $O_A(a), P_A(b)$ and $\vdash a \to \neg b$, then $O_A, P_A \vdash \bot_{O_A, P_A}$.

This captures the spirit of (5) by suggesting that an authority would be inconsistent if it imposed an obligation that contradicted a permission it had granted.

If we wish to formulate statements about the obligations and permissions of rational authorities, we could express them in terms of the negation of those deontic statements that would give rise to an inconsistency for that authority. From (7), it can be seen that a rational authority would not maintain both $O_A(a)$ and $P_A(\neg a)$.

Revisiting (6), we could consider the case where the obligations conflict. From Rule 2 we can obtain the following.

(8) If $O_A(a)$ and $O_A(b)$ and $\vdash a \to \neg b$, then $O_A \vdash \bot_{O_A}$.

From this it can be deduced that a rational authority would not impose conflicting obligations such as $O_A(a \wedge b)$ combined with $O_A(\neg a)$.

If we are in the business of formulating a normative code, then it is a good idea to maintain consistency by avoiding inappropriate combinations of obligations and permissions. To do so by considering (7) and (8) will give rise to a system of authority that complies with the spirit of the SDL theorems in (4) and (5).

5 Conclusions and Future Work

The paper presents an approach for capturing aspects of the semantics of obligations and permissions in a way that allows us to deal with unmet obligations and inconsistent codes of conduct without resorting to the machinery of paraconsistent logic, non-monotonic reasoning, multiple levels of obligation or the use of somewhat esoteric possible-worlds models. It does so by considering a failure to comply with an obligation as giving rise to a transgression. Such transgressions are more fine-grained in character than the universal sanction proposed by Prior (1958), and developed by Nowell-Smith & Lemmon (1960).

The treatment provides a fine-grained framework in which issues concerning negation and entailment, and various deontic puzzles can be considered without sometimes inappropriate solutions being imposed or required by a more conventional possible-worlds style account.

There are a number of outstanding issues to address, including: quantification; the syntax-semantics interface; and the interplay between obligations, permissions, and an agent’s plan of action. Consideration also has to be given to other deontic puzzles, and whether there are cases for which the system needs to be strengthened in order to accord with our intuitions about deontic reasoning. It may also be possible for the formulation of the theory to be simplified and expressed in different terms.

When it comes to the analysis of natural language deontic statements, it is apparent that a full account needs to consider the role of other aspects of interpretation — including context and focus — in determining their intended meaning, regardless of the underlying theory of deontic reasoning (Fox 2009). This can complicate the process of deciding what behaviour should be supported by the underlying deontic system.
It is necessary to produce a model of transgressions that can be used to demonstrate that the approach is formally coherent. This may also facilitate more extensive comparisons with alternative approaches, whose formulation sometimes relies on model-theoretic interpretations.

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