On the expressive power of some non-standard constituents

GQT has given rise to various new results concerning expressive power of NLs. One such result is announced in Nam (2005). He claims to have shown that NLs require for their interpretation, in finite universes, the full power of type $\langle 2 \rangle$ quantifiers, i.e. the full set of functions from binary relations to truth-values. The reason Nam gives is that finite meets and joins of sequences of two type $\langle 1 \rangle$ quantifiers lie outside of the set of reducible type $\langle 2 \rangle$ quantifiers.

In this talk I contest Nam's result on empirical grounds. I show that his result is based on overgeneralised empirical data and does not take into account some constraints on gapped constructions. I propose a specific semantic constraint on gapping and show how it can be used to show that some type $\langle 2 \rangle$ quantifiers are not denotable by gapped constructions.

Nam considers type $\langle 2 \rangle$ quantifiers induced by a sequence of two type $\langle 1 \rangle$ quantifiers. Any sequence of two type $\langle 1 \rangle$ quantifiers Q_1Q_2, where Q_1 corresponds to subject NP and Q_2 to direct object NP, induces a type $\langle 2 \rangle$ quantifier F in the following way: $F(R) = Q_1Q_2(R) = Q_1(Q_{acc}(R))$, where Q_{acc} is the accusative case extension of Q_2 defined as follows: $Q_{acc}(R) = \{x : Q_2(xR) = 1\}$ (where $xR = \{y : xRy\}$ and $Rx = \{y : yRx\}$.

Such type $\langle 2 \rangle$ quantifiers induced by case extensions are (Fregean) reducible. However, (Keenan 1992), meets and joins of such quantifiers need not be reducible. Given that NLs exhibit constructions whose interpretations necessitate the use of meets or joins of quantifiers induced by sequences of two type $\langle 1 \rangle$ quantifiers (this happens in the case of gapped sentences) we need finite meets and joins of such quantifiers. Nam shows that meet/join closure of such quantifiers forms an atomic Boolean algebra identical to the algebra of all type $\langle 2 \rangle$ quantifiers.

It is essential for Nam's proof to consider the closure of pairs Q_iQ_j under *arbitrary* joins and meets (pointwise) since only such arbitrary closure is closed under Boolean complements: the closure of meets and joins of reducible type $\langle 2 \rangle$ quantifiers satisfying some specific conditions may not result in the full algebra of type $\langle 2 \rangle$ quantifiers.

I take it that Nam claims that the full compositional semantics of a NL must have recourse to all type $\langle 2 \rangle$ quantifiers only if it is supposed to interpret *directly*, on "the surface", expressions of this language since gapped constructions can be interpreted by sentential constructions. I take it also that his claim is essentially based on the existence of gapped constructions in NLs. This means that I am not going to show that NLs do not need the full power of type $\langle 2 \rangle$ quantifiers for their interpretation but only to show that such a necessity does not follow from the existence of gapped constructions.

Finally I take it that Nam's claim is related to the existence of the conjunctions or disjunctions of pairs of NPs denoting type $\langle 1 \rangle$ quantifiers and not other expressions as in (1):

(1) Leo kissed Lea and nobody else kissed anybody else.

Interpretation of the above example does not involve just meets or joins of sequences of two type $\langle 1 \rangle$ quantifiers and *nobody else* does not denote an extension of a type $\langle 1 \rangle$ quantifier.

The last precision concerns the main logical connective occurring in gapped constructions. Since we are interested in meets and joins we consider that all gapped constructions refer to Boolean coordinations which can always be expressed as conjunctions or disjunctions of a sentence with a gapped constituent. This precision concerns only the main connective of the gapped construction. Since the gapped constituents probably have the Boolean structure, their complex Boolean compound can explicitly contain another connective as in (3a) and (3b):

(3a) Leo kissed Lea and either Bill Sue or Sam Mary.
(3b) Leo kissed Lea and neither Bill Sue nor Sam Mary.

In (3a) and (3b) the main connective is the conjunction *and*. The gapped constituent in (3a) is composed of sequences of NPs conjoined by *either ... or* and in (3b) the gapped constutuent contains sequences of N Ps conjoined by *neither... nor*.

Given above remarks Nam's claim that I am going to criticise can be formulated as follows: any type $\langle 2 \rangle$ quantifier (which is a meet or a join of type $\langle 1 \rangle$ quantifiers) over a finite domain can be denoted by a complex expression (let's say in English) containing gapped, possible Booleanly complex, constituents. We can even add, given the possibility of "transferring" the sentential negations to quantifiers and to the corresponding gapped constituent, that only proper names, conjunctions and disjunctions, and their negations, should be involved.

My disagreement with Nam concerns the question of whether all the meets and joins of all two-member sequences of type $\langle 1 \rangle$ quantifiers interpret (directly) some gapped sentences. In order to show that this is not the case it is enough to exhibit one particular constraint on possible NPs which can form non-standard constituents in gapped sentences and which implies the impossibility for some type $\langle 2 \rangle$ quantifier to be denoted by a gapped construction.

Gapping is a delicate phenomenon and judgements on it are subtle and it is not obvious that all languages have it (Tang 2001). In some cases additional pragmatic factors may disturb judgements of grammaticality. It has been early observed, however, that negations have a particular status in gapped sentences and that their use is strongly constrained. Since in the context of quantifiers it is preferably to use the notion of monotonicity instead of negation, we can state roughly a semantic constraint on the noun phrases which can be used in gapped constructions in the following way: if we have a meet or a join of two sequences of type $\langle 1 \rangle$ quantifiers which interpret a gapped sentence then the second member of the meet (or join) has to take into account the polarity or the type of monotonicity of the first member. Thus consider:

(4a) Nobody/no student kissed Lea but Bill/some teachers kissed Sue.
(4b) *Nobody/*no student kissed Lea but Bill/some teachers Sue.
(5a) Leo kissed no student and Bill no teacher.
(5b) *Leo kissed no student and Bill Sue.
(6a) Every student kissed Leo and every teacher Lea.
(6b) *No student kissed Lea and/but every teacher Sue.
(7a) Every student kissed Lea but not every teacher kissed Sue.
(7b)*Every student kissed Lea but not every teacher Sue.
(8) Neither no teacher kissed no student nor Bill Sue.

The above examples indicate dependence of gapping on monotonicity of quantifiers denoted by various NPs. Since the negation of the argument (of the binary relation) of a reducible type $\langle 2 \rangle$ quantifier is related to the postnegation of the second quantifier in the sequence it may happen that monotonicity of quantifiers is decisive in the choice of the connector in the gapped construction. This fact can be illustrated with sentences where only proper names are used:

(9) Leo did not kiss Lea and/but Bill kissed Sue.
(10) Leo did not kiss Lea and Bill did not kiss Sue.
(11a) ?Leo did not kiss Lea and Bill Sue. (11b) Neither Leo kissed Lea nor Bill Sue.

One observes that (9) does not have a gapped counterpart and the preferable gapped form of (10) is (11b) and not (11a).

Given the above examples in which gapping is not possible we can formulate the following more precise constraint on gapping: if the first member (a sequence of two type $\langle 1 \rangle$ quantifiers) of the meet or join induces a monotone decreasing type $\langle 2 \rangle$ quantifier then the second sequence must also induce a monotone decreasing quantifier. This constraint involves monotonicity of reducible type $\langle 2 \rangle$ quantifiers and thus we have to see how the monotonicity of a reducible type $\langle 2 \rangle$ quantifier is determined by the monotonicities of type $\langle 1 \rangle$ quantifiers whose iteration composes it. This dependence is expressed by the following propositions:

Prop 1: If Q_1 and Q_2 are monotone increasing then Q_1Q_2 is monotone increasing.
Prop 2: If Q_1 is monotone decreasing and Q_2 is monotone increasing then Q_1Q_2 and Q_2Q_1 are monotone decreasing.
Prop 3: If Q_1 and Q_2 are monotone decreasing then Q_1Q_2 is monotone increasing.

The above propositions express in fact necessary conditions for a reducible type $\langle 2 \rangle$ quantifier $F = Q_2Q_1$ to be monotone. The following facts also will be used: if F_1 and F_2 are monotone increasing (decreasing) then $F_1 \vee F_2$ and $F_1 \wedge F_2$ are monotone increasing (decreasing). Furthermore a meet or a join of a monotone increasing quantifier and of a monotone decreasing quantifier may be neither monotone increasing nor monotone decreasing quantifier.

It can now be easily checked that the proposed constraint applies to various examples above. For instance, given that *nobody* and *no student* denote monotone decreasing quantifiers and proper nouns and the NP *some teachers* denote monotone increasing quantifiers, in (4a) the first sequence of NPs denotes a decreasing type $\langle 2 \rangle$ quantifier and the second sequence denotes an increasing quantifier. Hence, given the constraint, the impossibility of (4a).

Notice that if we interpret conditional sentences by material implication then probably our constraint also explains the impossibility of gapping in conditional sentences; if (12a) is interpreted by (12b) then our constraint explains the marked ungrammaticality of (12c):

(12a) If Leo kissed Lea then Sam kissed Sue.
(12b) Either Leo did not kiss Lea or Sam kissed Sue. (12c) *If Leo kissed Lea then Sam Sue.

The proposed constraint does not say anything about gapped constructions in which corresponding quantifiers are neither monotone increasing nor monotone decreasing as in (13):

(13a) Only Leo kissed Lea and only Bill Sue. (13b) ?Only Leo kissed Lea and Bill Sue.

In (13a) the quantifiers denoted by *Only Leo* and *Only Bill* are not monotone (increasing or decreasing) and consequently their iterations do not form monotonic quantifiers and still the sequence is grammatical.

Our purpose is not to fully specify constraints on gapping. I want to indicate that some constraints are enough to show that not all type $\langle 2 \rangle$ quantifiers are denotable in NLs. The proposed constraint and one of its symmetric version that I am going to discuss now are sufficient for that purpose.

Can the above constraint can be symmetrically inversed: do we have gapped sentences in which the first quantifier is monotone increasing and the second is monotone decreasing? The following example shows that such symmetric constraint does not hold in general:

(14a) Leo kissed somebody/some student and/but Bill nobody/no teacher.
(14b) *Bill kissed nobody and Leo somebody.

So the constraint saying that if in a gapped construction the first induced type $\langle 2 \rangle$ quantifier is monotone increasing then the second should be also monotone increasing does not hold in general. If we consider, however, gapped constructions in which only proper nouns occur and in which the gapped constituent is simple (consisting of one pair of NPs which both are proper names), then this symmetric version of the constraint holds. Compare:

(15a) *Leo kissed Lea and/but not Bill Sue.
(15b) *Every student kissed Lea and/but not Bill Sue.
(16a) Leo kissed Lea and/but neither Bill Sue nor Sam Pat.
(16b) *Leo kissed Lea and Bill Sue or/and nobody Pat.

Thus we cannot negate a simple gapped constituent if the first conjunct of the gapped construction expresses a monotone increasing quantifier. Notice that examples (15) should be distinguished from the one in (16). In (16a) the second conjunct of the gapped construction is a complex gapped constituent. This complex constituent corresponds to a monotone decreasing type $\langle 2 \rangle$ quantifier since it is a meet of two decreasing quantifiers. Thus when in a gapped construction the first conjunct induces an increasing quantifier the gapped constituent cannot be syntactically atomic and express a decreasing quantifier. We observe also that when the gapped constituent is syntactically complex, that is a coordination of many sequences of two NPs then any such sequence must induce a quantifier of the same monotonicity. (cf. 16b).

So the specific version of the constraint to be used to show the falsity of Nam's claim is as follows: if the first conjunct of a gapped construction expresses a monotone increasing reducible type $\langle 2\rangle$ quantifier then the second conjunct, the gapped constituent, cannot be atomic and denote monotone decreasing quantifier.

I show now how some type $\langle 2\rangle$ quantifiers are not denotable by gapped constructions. The quantifiers I consider are specific atoms of the algebra of all type $\langle 2\rangle$ quantifiers that is functions which are true of just one binary relation. So any atom of the algebra of type $\langle 2\rangle$ quantifiers has the form F_R, where R is the only binary relation of which F_R is true. The relation R in F_R is called the *index* of the atomic quantifier F_R. We will not consider all atomic quantifiers but only those which are indexed by co-atomic relations, that is relations which contain every but one ordered pair. Thus we will consider atomic quantifiers of the form $F_{\{\langle a,b\rangle\}'}$, for any $a, b \in E$ (where E is a finite universe and $\{\langle a, b\rangle\}'$ denotes the set composed of all elements of $E \times E$ except the pair $\langle a, b\rangle$). Such atomic quantifiers are nor reducible (Keenan 1992) and we show that they are not denotable by gapped constructions.

Any effability claim concerning NPs implies that it is possible to name in some way various objects in the universe. In our case we suppose for simplicity that we can name any object in the finite universe, that is for any finite set of objects we can find a set of individual denoting NPs which refer to these objects. Consider now the atomic quantifier $F_{\{\langle a,b\rangle\}'}$. This quantifier is true of just the relation which contains all pairs except the pair $\langle a, b\rangle$. As any atomic quantifier it can be represented as a meet of pairs consisting of two ultrafilters or pairs consisting of a complement of an ultrafilter and an ultrafilter. Ultrafilters are denotations of proper nouns. The atomic quantifier $F_{\{\langle a,b\rangle\}'}$ is defined as in (17) (where I_x is the ultrafilter generated by x):

(17) $F_{\{\langle a,b\rangle\}'} = (\bigwedge_{x\neq a, y\neq b} I_x I_y) \wedge (\neg I_a I_b)$

It follows from (17) that $F_{\{\langle a,b\rangle\}'}(R) = 1$ iff $R = \{\langle a, b\rangle\}'$.

(17) indicates how to construct a sentence corresponding to the function $F_{\{\langle a,b\rangle\}'}$ applied to R denoted by a TVP. Its non-gapped counterpart is just a "long" conjunction composed of sentences of the form $NP_i VP NP_j$. Thus suppose that the universe E contains $n + 2$ elements and let the expression *PrN(a)* denote a, *PrN(b)* denote b and *PrN(x)*, with varying x, denote all other elements of universe and the VP denote R. The form of the non-gapped counterpart corresponding to the quantifier given in (17) applied to R, denoted by *VP*, is given in (18):

(18) *PrN(a) not VP PrN(b) and PrN(1) VP PrN(1)... and PrN(n) VP PrN(n)*

It follows from constraints on gapping that (18) does not have a gapped counterpart. The reason is the only "negative" conjunct it contains. It cannot be placed at the beginning of the gapped constructions since the remaining part induces a monotone increasing quantifier. Similarly it cannot constitute the gapped constituent (after the deletion of the verb) since unique ("atomic") sequences of proper nouns inducing a monotone decreasing quantifier are not possible in gapped constructions. Finally, it cannot be a part of a gapped constituent since all other parts of such a constituent should be "positive".

It is not quite a proof that the quantifier represented in (17) is not denoted by any expression in English. It just shows that gapping expressions of some natural form cannot denote it. This is enough to consider that the effability claim announced by Nam is not correct: natural languages, at least English, do not need all type $\langle 2\rangle$ quantifiers for their semantics. Consequently, gapped constructions do not force us to type $\langle 2\rangle$ effability.

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