The Tulip Project

a partial treatment of formal analogies

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plan

1. why “tulip”? 
2. why “partial”? 
3. why “treatment”? 

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• ancient use: ‘regularity’
• beginning of 19th century: exception to “sound laws”
• e.g., Hungarian *sirolm* > *siralom* ~ *siralm*- ⇒ *sátor* > *sátor* ~ *sátr*-
• de Saussure: back to the ancient view, analogy is the force that makes linguistic systems coherent
example

\[ A : B = C : ? \]

\[ \begin{array}{ccc}
A & \rightarrow & B \\
\downarrow & & \downarrow \\
C & \rightarrow & ?
\end{array} \]
analogy

example

\[ A : B = C : ? \]

\[ \text{son} \rightarrow \text{sons} \]

\[ \text{sin} \rightarrow ? \]
analogy

example

\[ A : B = C : ? \]

- transformation \( son \leadsto sons \)
- transformation \( son \leadsto sin \)
- composition of the two: \( son \leadsto sins \)

Lepage (1996)
Lepage was the first to offer a formal treatment of analogy, based on string operations (deletion and insertion, just like in the calculation of Levenshtein distances), and he has used it for finding regularities in large corpora.

However, this method is difficult to generalize to more sophisticated representations and operations; I felt that a more principled and abstract way of approaching analogy is called for.

In what follows, I will first show, starting from this simple string example, what analogy means at the most abstract level.
An analogy is illustrated below:

\[ A : B = C : ? \]
analogy

\[ A : B = C : ? \]

son \[\rightarrow\] sons

sin \[\downarrow\] \[\downarrow\]

sins
$A : B = C : ?$

son  
\[\downarrow\]  
\[\sin\]  
\[\downarrow\]  
\[\textit{sins}\]

\[\rightarrow\]

\[\textit{sons}\]
A : B = C : ?

son \ son = -s

son \ son = -i-

son \ sons \ sin = s \ -n

? = -s \cup -i \cup s \ -n = sins

metaphorically

analogy
\[ A \cap B \cap C \]
It seems that the “tulip” is a suitable set theoretical metaphor, but how will we make it operational? What should be the members of our sets? They should be something like “ingredients” or “properties” of representations, but this is too general, and the calculi of such entities is often non-trivial.
why partial?

<table>
<thead>
<tr>
<th>things i cannot do (and maybe do not want to)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• reduplication: <em>tami</em> : <em>tatami</em> = <em>paya</em> : ?</td>
</tr>
<tr>
<td>• metathesis: <em>top</em> : <em>pot</em> = <em>lead</em> : ?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>things i cannot do (but would like to)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• sub- (auto-) segmental phenomena: <em>teeth</em> : <em>teethe</em> = <em>belief</em> : ?</td>
</tr>
<tr>
<td>• semantic phenomena: <em>sleep</em> : <em>slept</em> = <em>go</em> : ?</td>
</tr>
<tr>
<td>• true analogy is not based on individual examples, but legions of them, with varying frequencies</td>
</tr>
</tbody>
</table>
elements of a solution

- approach 1: strings represented as sets of binary trees with ordered branches ("left" and "right") with constraints as leaves
- approach 1: strings represented as partial orders over subsets of a set with an equivalence classification (corresponding to features)
- difference, intersection and "union" (combination) are defined accordingly
approach 1: trees

son

sin

sons
approach 1: trees

intersection

\[
\begin{array}{c}
s \quad \ast \quad n \\
\ast \quad n \quad s \quad \ast
\end{array}
\]

difference

\[
\begin{array}{c}
\ast \quad s
\end{array}
\]

difference

\[
\begin{array}{c}
\ast \quad \ast \\
i \quad \ast \quad \ast \quad i
\end{array}
\]

union

\[
\begin{array}{c}
s \quad s \\
\ast \quad i \quad n \quad s \quad i
\end{array}
\]
**brief description**

- the universe $\mathcal{U} = \langle \mathcal{E}, \equiv \rangle$ (entities with an equivalence relation $\equiv \subseteq \mathcal{E}^2$, representing features)
- string constraints $\langle E, \leq \rangle$, with $E \subseteq \mathcal{E}$, a partial ordering $\leq$ over $E$
- for a simple string, $x_1^{(s)} \leq x_2^{(o)} \leq x_3^{(n)} \leq x_4^{(s)}$

**the tricky part: operations**

- they are defined over sets of constraints
- maps from “sons” to “son” and from “sin” to “son”
- difference: unmapped part; intersection: covered by both ranges
- linearization also yields sets of strings
- modelling sub- and autosegmental representations (partial order of timing points that start or end an autosegment)
Köszönöm a figyelmét!

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